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ALLOCATION OF SHELF SPACE: A CASE STUDY OF REFRIGERATED JUICE PRODUCTS IN GROCERY STORES

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Allocation of Shelf Space:

A Case Study of Refrigerated Juice Products in Grocery Stores*

The amount of shelf space allocated to a product in a store has an opportunity cost, knowledge of which allows determination of the most profitable allocation of space between alternative products competing for the store's limited shelf space (Cairns, Cox). Factors to consider in determining the optimal allocation include the per unit profit of each product and the demand levels for the products which, in turn, can expected to be dependent on such factors as prices, consumer income and preferences. Preferences, in turn, may be related to such factors as advertising, demographic variables and shelf space which by itself may be a form of advertising indicating popularity of products and putting them on tops of consumers minds.

Shelf may also affect demand by reducing consumer search costs. Regardless, a product's sales can be expected to be dependent on the amount of shelf space allocated to the product, as well as characteristics of the space such as location (Pauli and Hoecker, Kotzan and Evanson, Frank and Massy, Anderson, Wilkerson, Mason and Paksoy, Corstjens and Doyle). In this case,the optimal allocation of shelf space would depend, in part, on the marginal impacts of shelf space on product demands. Other factors such as the cost of product procurement, storage, insurance, spoilage and out-of-stock costs may also be relevant, in which case they should be included in the allocation problem.

In this paper, we examine a line of grocery store products with similar cost structures. We assume these costs will not change due to a reallocation of shelf space. Hence, we focus on demand and its relationship with shelf space. The paper proceeds by examining more closely the question of optimal shelf space

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using a simple mathematical model. Then an empirical study of refrigerated juice and juice-drink product shelf space is made to illustrate how the model works. The results indicate where store management might look, and perhaps experiment with, in allocating shelf space to maximize profits.

Model

A profit maximization model is used to reveal the basic aspects of the shelf space allocation problem. The problem is to allocate a limited amount of shelf space between n products such that store or store department profits are maximized. Letting S, s(i), p(i), w(i), q(i), x be the total fixed shelf space; shelf space allocated to product i; retail price of product i; wholesale price of product i; quantity demanded of product i; and a vector containing the n retail prices, consumer income, advertising and other demand explanatory variables, respectively, the maximization problem can be written

(1) maximize $V=m'q$, subject to $S=1's$,

where $m'=(p(1)-w(1),\ldots)$, $q=(q(1),\ldots)$, 1 is a unit vector, and $s'=(s(1),\ldots)$. The term m(i) measures the profit per unit of product i, with retail and wholesale prices assumed to be fixed by competitive forces. The demands for products can further be written as functions of x and s, i.e., $q=f(x,s)$. Additional constraints such as lower and upper bounds and/or nonnegativity on s(i) might also be included in specifying problem (1) (e.g., see Corstjens and Doyle).

The Lagrangian for (1) can be written as $L=m'q+r(S-1's)$, where r is the Lagrangian multiplier, and the first order conditions are

(2a) $dL/ds(i) = m'*dq/ds(i) - r = 0, \quad i=1,\ldots,n.$

(2b) $dL/dr \quad = S - 1's = 0,$

where, $dq/ds(i)=(dq(1)/ds(i),\ldots,dq(n)/ds(i))$, with, dy/dz, in general, being the partial derivative of y with respect z.

Approximating demand by the double log function $q(i)=A(i)*s(1)**e(i,1)*,\ldots,*s(n)**e(i,n)$ where $e(i,k)$ is the elasticity of demand for product i with respect to shelf space for product k, and $A(i)$ is a function of x, (2a) can be written as

(3) $B(i)=r*s(i),$

where $B(i)=m(1)*q(1)*e(1,i)+\ldots+m(n)*q(n)*e(n,i)$ and (2a) has been multiplied through by $s(i)$.

Summing (3) over i, the solution for the Lagrangian multiplier can be written as

(4a) $1'B=r*1's,$

or

(4b) $r=1'B/S,$

where $B=(B(1),\ldots)$ and we have used the constraint $S=1's$. One can show that r is marginal profit obtained by relaxing the shelf space constraint by a unit.

Substituting (4b) into first-order conditions (3), the solution for $s(i)$ can be written as

(5) $s(i)/S = B(i)/1'B.$

Solution (5) indicates that the share of shelf space allocated to product i is product i's share contribution to the profit term 1'B. The term B(i) can be interpreted as profit resulting from changing product i's allocation of shelf space from zero to s(i), i.e., B(i)=m(1)*q(1)*e(1,i)+...+m(n)*q(n)*e(n,i) or B(i)=(m(1)*dq(1)/ds(i)+...+m(n)*dq(n)/ds(i))*s(i), and 1'B is the overall profit of shelf space S.

Case 1: Zero Cross-Shelf-Space Elasticities and Constant Elasticity of Substitution

As (1) through (5) shown, in general, optimal allocation of shelf space can be expected to be a function of both own-shelf-space elasticities (e(i,i)'s) and cross-shelf-space elasticities (e(i,k)'s). In our empirical analysis, both own and cross shelf space effects are considered. However, because the cross shelf space effects are not statistically different than zero, such cross effects are ignored below to focus on the own shelf space effects.

Neither solution (4b) or (5) are in reduced form as the s(i)'s are on the right sides of the solutions. To obtain an explicit closed-form solution, further structure needs to be given to the problem. For example, suppose all e(i,i) are the same, i.e., e=e(i,i). This is the constant elasticity of substitution (C.E.S.) assumption examined in demand and production models (e.g.,Varian). In this case, the shelf space solution can be written as s(i)/S=C(i)/1'C, where C(i)=(m(i)*A(i))**k, k=1/(1-e), and C=(C(1),...).[1] We assume that the shelf space elasticities are in the zero-one interval (0<e<1), assuring that the second order conditions are meet (this seems to be a reasonable assumption given stores have not grossly under or over allocated shelf space).

Although not in reduced form, (5) can be used to find a solution for s(i), using some iterative procedure (of course, this assumes that the second order conditions are fulfilled so that a solution exists). For example, initial values for the s(i)'s can be substituted into the right-hand-side of (5) to obtain updated s(i)'s which can then be substituted into (5) to obtain further updates. This procedure can be repeated until convergence is achieved, i.e., the values of s(i) are the same on both sides of (5).

Case 2: Constant Elasticity of Substitution and Constant Percentage Markups

With some further simplifying assumptions, solution (5) can also be used to examine summary data on shelf space and sales, excluding detailed demand estimates. For example, suppose shelf space for a department in a store is fixed and similarity of products allows the C.E.S. approximation to be used. In this case, (5) becomes

(5a) s(i)/S=m(i)*q(i)/m'q.

If all retail prices are approximately marked up by the same percentage, i.e., p(i)=(1+g)*w(i),g>0, then s(i)/S=p(i)*q(i)/p'q, where p'=(p(i),...). That is, a product's share of department retail dollar sales equals the share of department shelf space that should be allocated to the product for profit maximization. From this result, we can also see that each product's average dollar sales per unit of shelf space is the same for an optimal allocation, i.e., p(i)*q(i)/p'q=s(i)/S or, after rearranging, p(i)*q(i)/s(i)=p'q/S for all i.

Case 3: Constant Elasticity of Substitution and Constant Absolute Markups

Similarly, if the C.E.S. assumption is maintained and all products have the same absolute markup, m(i)=m, then

(5b) $s(i)/S= q(i)/1'q$,

or the share of department quantity sales accounted for by a product equals its share of shelf space. Of course, one would prefer to have knowledge of per unit profits, demand levels and demand impacts of shelf space, so that (5) could be applied more precisely.

Application

Data on shelf space, and dollar and gallon sales for juice and juice-drink products in refrigerated departments of grocery stores were examined for consistency with the above theoretical results. The data were provided by Nielsen Marketing Research and are for U.S. grocery stores doing $4 millon or greater business, for the week ending March 12,1994. The analysis takes the total shelf space allocated to the juice and juice-drink products as given.[2] Shelf space was measured by number of visible product facings (given juice and juice-drink product similarity, the number of linear inches per facing is expected to be about the same across the products studied).

The product with the largest sales and shelf space in the refrigerated juice and juice-drink department is orange juice (OJ). Focusing on OJ, all other juices and juice-drinks were aggregated into a remaining juice (RJ) category, and the question whether the OJ-RJ shelf space allocation is consistent with our theoretical results was examined. Table 1 shows the basic data. The shares of department dollar and gallon sales accounted for by OJ are substantially greater than the corresponding shelf space share---OJ had 51% of the department facings versus 72% of the department dollar sales and 67% of the gallon sales. Does this suggest that shelf space is under allocated for OJ? Based on result (5), it is possible that profit per unit of product, demand level and impact of shelf space on demand could be such that an outcome as in Table 1 is consistent with profit maximization. However, with similarity of juice-and-juice-drink products, the C.E.S. specification and across-product equality of mark-ups (either in percentage or absolute terms) may be reasonable approximations. In this case, OJ appears to have less shelf space than would be optimal, based on discussion in the Model section.

Constant Elasticity of Substitution and Constant Percentage Markups

To show that OJ has less than optimal shelf space for the C.E.S. model, first note that, for equal percentage mark-ups across products, profits V are

V=k1*((p(1)*q(1)+p(2)*q(2)+...),

where k1 is some percentage. Given the C.E.S. parameter assumptions, the change in profits for a reallocation of shelf space is

dV=k1*e*(avg(1)*ds(1)+avg(2)*ds(2)+...),

where avg(i)=p(i)*q(i)/s(i) or average retail dollar sales per unit of shelf space for product i; and ds(i) is the change in product i's shelf space. Given the shelf space constraint, note ds(1)+ds(2)+...+ds(n)=0 or ds(1)=-ds(2)-ds(3)-

...-ds(n). Substituting the latter result into dV, we find

$$dV=k1*e*((avg(2)-avg(1))*ds(2)+(avg(3)-avg(1))*ds(3)+...).$$

That is, if product 1 were OJ, profits could be increased by increasing OJ's shelf space and decreasing the shelf space for the remaining products, since, as shown in Table 1, OJ has the greatest average retail dollar sales per facing of shelf space, i.e., avg(i)-avg(1)<0 for i=2,3,...,n, implying ds(2) through ds(n) should be negative for an increase in profit.

Demands for OJ and RJ as Functions of Shelf Space

Cross sectional data were further analyzed to determine whether OJ might have less than optimal shelf space. In sampling U.S. grocery stores, Nielsen has divided the U.S. into 51 regions. For each of these regions, data on shelf space, dollar and gallon sales, consumer income and population are available. From this data, the logs of per capita gallon sales of OJ and RJ were calculated and used as dependent variables in demand regressions . The explanatory variables for each demand regression were the logs of prices of OJ and RJ; the logs of the average OJ and RJ facings per store; the log of per capita income; an age variable, defined as the percentage of the population over 34 years old; and two regional dummy variables, one for the Northeast and the other for the West.[3] Descriptive statistics for the data are shown in Table 2.

All explanatory variables were treated as predetermined and independent of the demand equation error terms. If the time interval of an observation were not a week, this assumption might be inappropriate for the price and shelf space variables. However, given such a short time period, prices and shelf space are expected to be essentially fixed by grocery store plans, including promotional specials that typically last for a week or more and provide consumers with some type of deal which does not change over the period offered. In addition, the facings-per-store variables reflect average stocked shelf space; out-of-stocks are assumed to be similar across the regional observations.[4]

Preliminary regression results suggested that the cross-facings-per-store effects are not significant in explaining OJ and RJ demands. Initially, the demand equations were estimated by ordinary least square (OLS), and t tests were made to test whether the cross-facings effects were zero. For each equation, the cross-facings effect was not different than zero at any reasonable level of significance (the cross-facings t values were .87 and .46 for the OJ and RJ equations, respectively). A quasi-likelihood ratio test (Gallant and Jorgenson) was also used to jointly test the significance of the two cross-facings effects based on seemingly unrelated regression (SUR) estimates. The SUR test result confirmed the OLS result, with the SUR asymptotic chi-square test statistic taking a value of.76 with two degrees of freedom.

SUR estimates for the OJ and RJ demand equations, with the cross-facings-per-store effects restricted to zero, are shown in Table 3. All parameter estimates are significant at the 5 percent level, except that for the Northeast dummy variable in the RJ equation. As the demand equations are in double log form, the parameter estimates, except those for the age and regional dummy variables, are elasticities. The own-price elasticities for OJ and RJ are -.8 and -2.5, respectively, indicating an inelastic OJ demand and an elastic RJ demand. The price of RJ has a negative or complementary effect on OJ demand, while the price of OJ has a positive or substitute effect on RJ demand. The income elasticity for each demand equation is positive, indicating normal type goods. The Northeast and West dummy variable effects are positive and negative, respectively, in each demand equation. The effect of an older age population is positive in both equations. The elasticity of OJ demand with respect to OJ facings per store is .48, indicating that a 1.0 percent increase in OJ facing per store would increase per capita gallon sales of OJ by .48 percent. Similarly, the elasticity of RJ demand with respect to the own facings variable is .55. That is, shelf space for RJ has a somewhat larger own effect than that for OJ. Based on equation (5), this result by itself favors RJ in the allocation of shelf space. However, Equation (5) also shows that mark-ups and demand levels, the latter which are also dependent on shelf space, are important in determination of optimal shelf space.

Optimal Shelf Space Allocation

The facings-per-store elasticity estimates in Table 3 were used to solve equation (5) for optimal allocation of shelf space at the U.S. sample mean.[5] Alternative mark-ups were assumed based on data on OJ and RJ mark-ups in 14 grocery stores in the Midwest. Four scenarios were examined. On average, the mark-up as a percentage of the retail price was 28 percent for OJ versus 32 percent for RJ in the Midwest stores. This set of mark-ups, plus three other sets, with larger mark-ups for RJ and smaller mark-ups for OJ (26 percent for OJ versus 34 percent for RJ, 24 percent for OJ versus 36 percent for RJ, and 22 percent for OJ versus 38 percent for RJ) were examined. The solutions to the latter three sets of mark-ups, which, of course, favor allocation of shelf space towards RJ, are helpful in determining whether OJ shelf space is under allocated. The levels of OJ and RJ demands, which are dependent on prices, income and shelf space, are also important in determination of optimal shelf space. The solution to equation (5), brings together the assumed mark-ups, estimates of the facings-per-store elasticities, and estimates of demand levels.

Table 4 shows the simulation results. Based on the average mark-ups for the sample of 14 stores, OJ would receive nearly 80 percent of the refrigerated juice and juice-drink department facings with total department sales in the United States increasing by about 3 million dollars per week or 6.2 percent. With the estimated facing elasticities being less than unity, average dollars per facing decrease for OJ, and increase for RJ and the department, as more shelf space is allocated to OJ and less to RJ.

At the present allocation in Table 1, the gain in profit resulting from allocating one more facing to OJ is the OJ mark-up (.28) times the OJ dollars per facing (20.07) times the OJ facings elasticity (.48) or $2.70, while the loss in profit resulting from allocating one less facing to RJ is the RJ mark-up (.32) times the RJ dollars per facing (8.17) times the RJ facings elasticity (.56) or $1.46; hence, the net profit gain is $1.24. These results indicate that, at present, OJ has a much smaller allocation of shelf space than suggested by our simple profit maximization model.

To better understand whether OJ's shelf space is really under-allocated, we turn to the alternative mark-up assumptions, favoring RJ. OJ's optimal share of department facings ranges from the 80 percent based on the average mark-ups to 61.6 percent based on a mark-up for RJ of 38 percent versus a mark-up for OJ of 22 percent. That is, even for a relatively high RJ mark-up compared to that for OJ, our profit maximization model indicates the present allocation of refrigerated shelf space provides OJ with too little space. There may be other reasons for the present allocation such as slotting payments by manufacturers to grocery stores to get their products on shelves, and, perhaps, a demand for variety by consumers which grocery stores may be responding to in order to maintain a base of customers. Further research might consider such possibilities, perhaps, in a profit maximizing framework as suggested here. However, given the present information, which has been analyzed here, it appears that the average grocery store could increase profits by increasing the amount of shelf space allocated to OJ.

Conclusions

The results of this study show how shelf space can be allocated between products in a profit maximizing framework. The basic problem and equations might also be useful for allocation of other fixed factors among alternative uses. For example, a fixed amount of advertising expenditure might be allocated between different products or different markets to maximize profits or sales---the question of an optimal allocation can be examined through the advertising impacts on demands and profits using the same approach outlined here.

To fully apply the results of this study, one needs estimates of product demands, including the impacts of shelf space or any other similar factor being examined, along with product mark ups or per unit profits. Demand interactions between shelf space and factors such as advertising and promotion might also be considered in an application. Given the foregoing information, shelf space allocation can be determined straightforwardly by (5) or some variant with an alternative demand specification, i.e., the double log demand specification in (5) need not be maintained. Using this approach, adjustments of shelf space for demand changes resulting from changes in the explanatory variables under consideration might be considered.

Footnotes

[1] To obtain the C.E.S. specification note

(a) s(i) = (e*m(i)*A(i)*s(i)**e / (e*m'q)) * S,

or

(b) s(i)**(1-e) = (m(i)*A(i) / m'q) * S;

or

(c) s(i) = ((m(i)*A(i) / m'q) * S)**(1/(1-e))),

or

(d) m(i)*A(i)*s(i)**e = m(i)*A(i)* ((m(i)*A(i) / m'q) * S)**(e/(1-e))),

or

(e) m(i)*q(i) = (m(i)*A(i))**(1/(1-e)) * (m'q)**(-e/(1-e))*S**(e/(1-e)),

or, summing over i

(f) m'q = 1'C * (m'q)**(-e/(1-e)) * S**(e/(1-e)),

where as defined previously C(i)=m(i)*A(i)**(1/(1-e)) and C=(C(1),...);

hence

(g) (m'q)**(1/(1-e)) = 1'C * S**(e/(1-e)),

or

(h) m'q = (1'C)**(1-e) *S**e.

Substituting (h) into (c) results in

(i) s(i) = (C(i) / 1'C) * S.

[2] A complete analysis would need to link the amount of shelf space in the refrigerated department to total store shelf space.

[3] The demand specifications can be viewed as approximations resulting from the average (per capita) consumer maximizing utility subject to a budget constraint

(average per capita income).

[4] The observed facings per store are stocked facings; out-of-stock facings were not measured. Given similar grocery store management practices and available product supplies across the United States, the percentage of total stocked-and-out-of-stock facings per store accounted for by stocked facings is expected to be about the same across regions.

[5] An intercept for each demand equation, calculated as the difference between the log of U.S. per capita gallon sales and the product of the facings-per-store elasticity times the log of U.S. mean facings per store, was used as the base for the alternative simulations.

Table 1. Dollar and Gallon Sales Versus Facings, in U.S. Refrigerated Juice and-Juice-Drink Departments, for Week Ending March 12, 1994[a].

	Orange Juice	Other Juices and Drinks	Dept. Total
Facings (000)	1718	1657	3375
% Dept. Tot. Fac.	50.9	49.1	100.0
Dollar Sales (000)	34476	13530	48006
% Dept. Tot. Dol.	71.8	28.2	100.0
Gallon Sales (000)	9266	4318	13584
% Dept. Tot. Gal.	68.2	31.8	100.0
Dollars/Facing	20.07	8.17	14.22
Gallons/Facing	5.39	2.61	4.02

[a] Data are for U.S. grocery stores, each with annual sales on all items of at least 4 million dollars.

Table 2. Descriptive Statistics for Regional Data, for Week Ending March 12,1994[a].

Variable	Mean	Std. Dev.
Weekly OJ per capita gallons	0.036	0.014
Weekly RJ per capita gallons	0.017	0.005
OJ price: $/gallon	3.704	0.613
RJ price: $/gallon	3.112	0.374
Annual Per capita income (000)	13.804	1.836
OJ facings per store	73.559	12.199
RJ facings per store	67.924	11.017
% population > 34	47.571	3.232
% sample in Northeast	19.608	40.098
% sample in West	31.373	46.862
% sample in Other Regions	49.020	50.488

[a] Data are for U.S. grocery stores, each with annual sales on all items of at least 4 million dollars.

Table 3. OJ and RJ Demands, SUR Estimates.

	OJ		RJ	
	Estimated		Estimated	
Variable	Coefficient	t-statistic	Coefficient	t-statistic
constant	-14.956	-5.42	-14.323	-4.98
OJ facings	.482	3.73		
RJ facings			0.555	4.42
OJ price	- .779	-2.42	1.137	3.38
RJ price	- .821	-2.45	-2.485	-7.14
income	1.056	3.11	0.856	2.44
age	.030	2.81	0.023	2.11
Northeast	.223	2.59	0.109	1.21
West	- .314	-3.13	-0.333	-3.23
	R-squared = .805		R-squared = .661	

Notes: The dependent variable is the log of per capita gallon sales; OJ and RJ facings are logs of facings per store; OJ and RJ prices are logs of prices; income is the log of per capita income; age is the percentage of the population over 34 years old; and Northeast and West are regional dummies variables, each equal to one if market is in region, other wise equal to zero.

Table 4. Optimal Percentages of Department Facings for Orange Juice and Other Juices & Juice Drinks, at the U.S. Sample Mean.

Scenario	Assumed % Mark Up OJ	Assumed % Mark Up RJ	Optimal % Department Total Facings OJ	Optimal % Department Total Facings RJ	Optimal % Department Total Dollars	
	%					
1	28.0	32.0	79.6	20.4	83.7	16.3
2	26.0	34.0	74.4	25.6	81.4	18.6
3	24.0	36.0	68.4	31.6	78.9	21.1
4	22.0	38.0	61.6	38.4	76.1	23.9
0	-	-	50.9	49.1	71.8	28.2

Scenario	Dollars/Facing OJ	Dollars/Facing RJ	Dollars/Facing Dept.	Total Dollars (000) OJ	Total Dollars (000) RJ	Total Dollars (000) Dept.
	$					
1	15.88	12.09	15.11	42660	8333	50993
2	16.44	10.94	15.03	41306	9438	50745
3	17.17	9.96	14.90	39667	10607	50274
4	18.13	9.13	14.68	37714	11819	49533
0	20.07	8.17	14.22	34476	13530	48006

Note: Scenario 0 is the average for the U.S. as shown in Table 1.

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